Current and Resistance

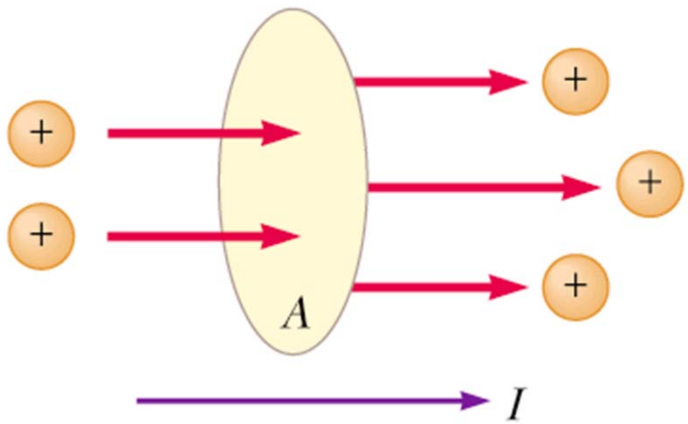
- Putting electrons in motion
- Electron movement through conductors
- Resistivity and Resistance – Ohm’s Law
- Electrical Power

Current

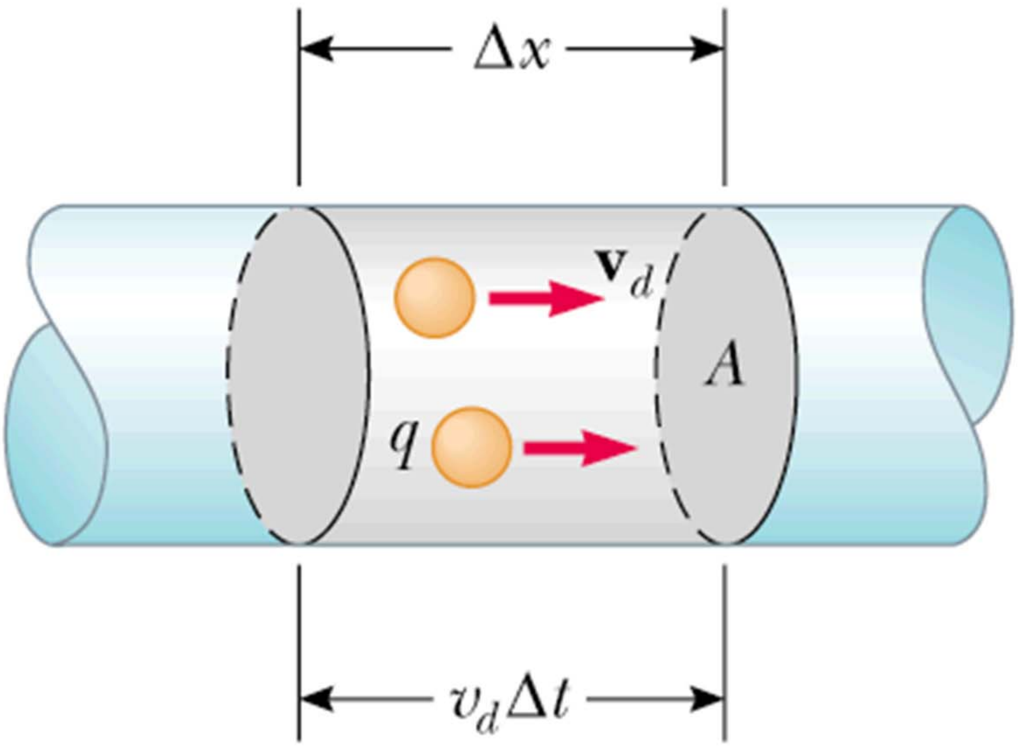
- Current is the flow rate of charges through a surface area.
- The direction of the current is the direction of the flow of positive charges.

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

$$I(t) = \frac{dQ}{dt}$$ *(C/s=Amperes)*



Microscopic Model



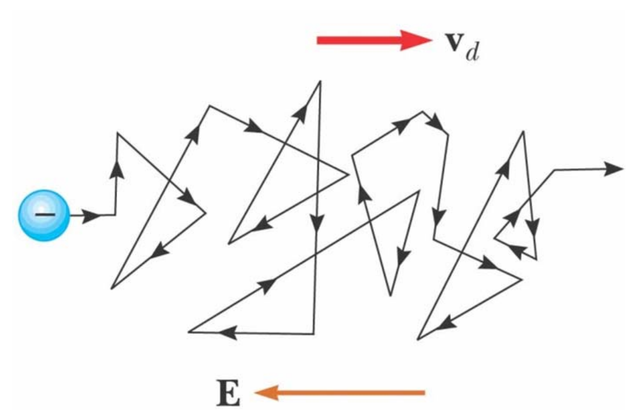
v_d : Drift Velocity

$$\Delta Q = (nV)q$$

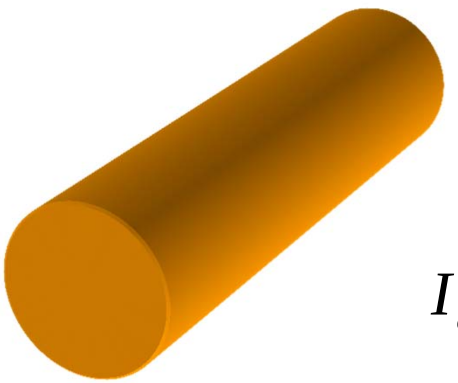
$$\Delta Q = (nA\Delta x)q$$

$$\Delta Q = (nAv_d\Delta t)q$$

$$I_{av} = \frac{\Delta Q}{\Delta t} = nqv_d A$$



Drift Speed In Copper Wire



A = 3.31 x 10^{-6} m^2
I = 10 A
ρ_{Cu} = 8.92 g/cm^3

$$I_{av} = \frac{\Delta Q}{\Delta t} = nqv_d A$$

$$10A = nv_d\left(1.6\times10^{-19}\right)\left(3.31\times10^{-6}\right)$$

$$V = \frac{m}{\rho} = \frac{63.5}{8.92} = 7.12cm^3/mol$$

m = 63.5 g/mol
N_A = 6.02 x 10^{23} at/mol
1 electron per Cu atom

$$n = \frac{N_A}{V} = \frac{6.02\times10^{23}}{7.12} = 8.46\times10^{22}el/cm^3 = 8.46\times10^{28}el/m^3$$

$$v_d = \frac{10}{\left(1.6\times10^{-19}\right)\left(3.31\times10^{-6}\right)\left(8.46\times10^{28}\right)} = 2.23\times10^{-4}m/s$$

Ohm's Law

Current Density:

$$J = \frac{I}{A} = nqv_d$$

(for a uniform current perpendicular to the cross-section)

$$\mathbf{J} = nq\mathbf{v}_d$$

General vector definition

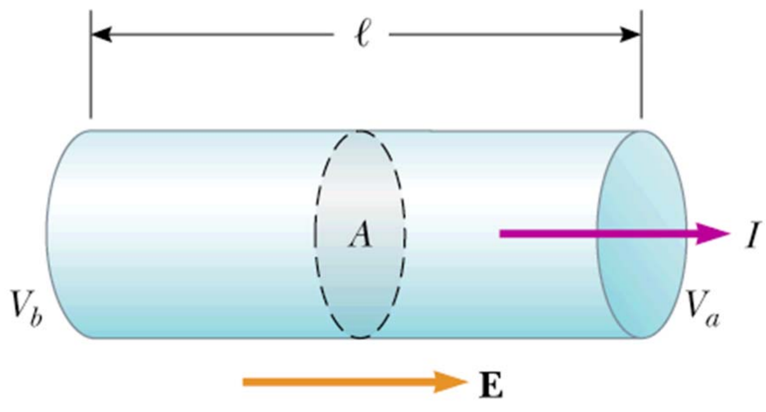
In some materials, the field applied and the current density are proportional.

Ohm's Law: $$\mathbf{J} = \sigma\mathbf{E}$$

σ: conductivity

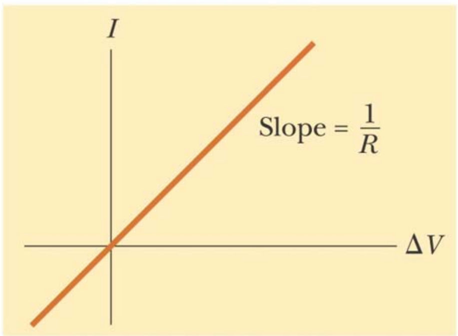
Most metals obey Ohm's Law

Resistance

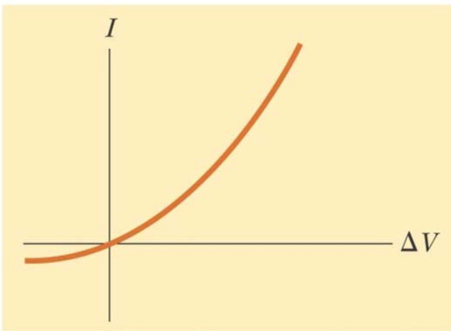


$$\Delta V = El = \frac{J}{\sigma} l = \left(\frac{l}{\sigma A}\right) I$$

$$R = \frac{l}{\sigma A} \equiv \frac{\Delta V}{I} \qquad (V/A = \Omega)$$



Metal



Semiconductor diode

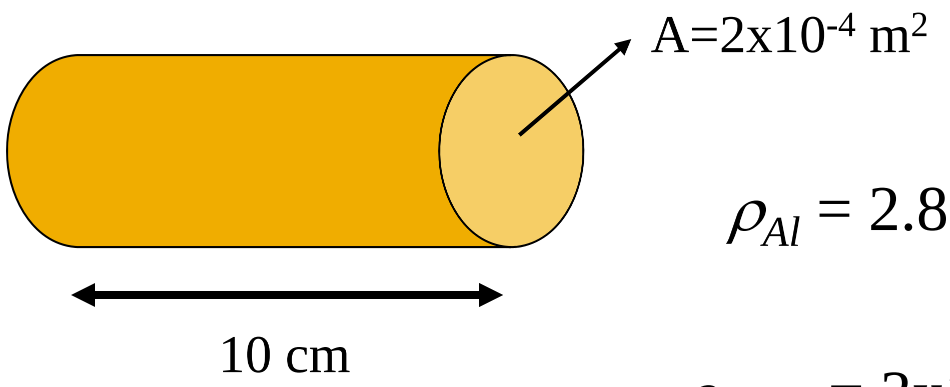
Resistivity

- Resistivity is the inverse of conductivity.
- Both are intrinsic properties of the material.
- Resistance is a also a function of the shape and size of the device.

$$\rho = \frac{1}{\sigma} \qquad R = \rho \frac{l}{A}$$

Material	$\rho\ (\Omega.m)$
Copper	$1.7\text{x}10^{-8}$
Gold	$2.44\text{x}10^{-8}$
Aluminum	$2.82\text{x}10^{-8}$
Silicon	640
Rubber	$\sim 10^{13}$

Resistance of a Conductor



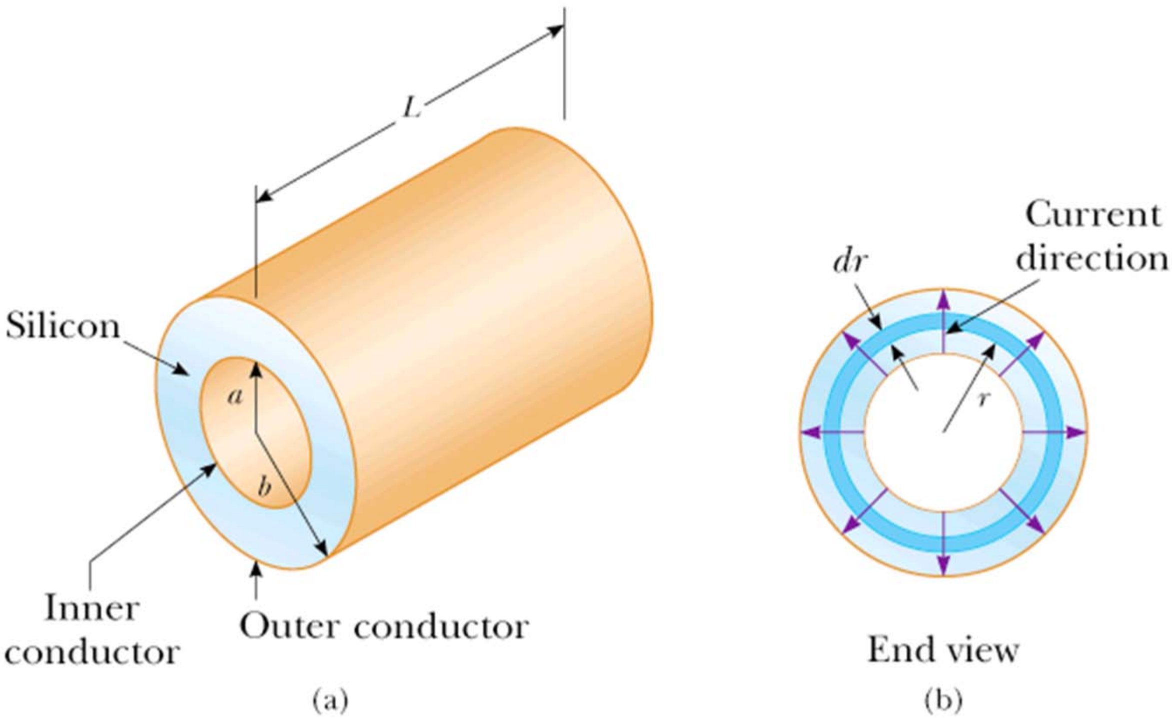
ρ_{Al} = 2.82x10^{-8} Ω.m

ρ_{Glass} = 3x10^{10} Ω.m

$$R_{Al} = \rho_{Al}\frac{l}{A} = \left(2.82\times10^{-8}\right)\left(\frac{0.1}{2\times10^{-4}}\right) = 1.41\times10^{-5}\Omega$$

$$R_{Glass} = \rho_{Glass}\frac{l}{A} = \left(3\times10^{10}\right)\left(\frac{0.1}{2\times10^{-4}}\right) = 1.5\times10^{13}\Omega$$

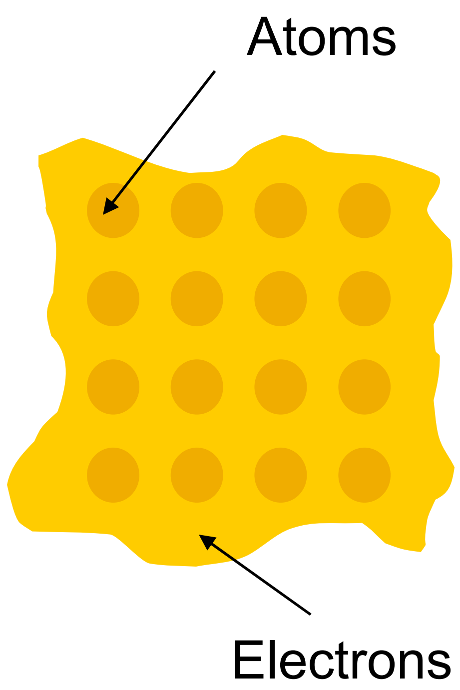
Radial Resistance of a Coaxial Cable



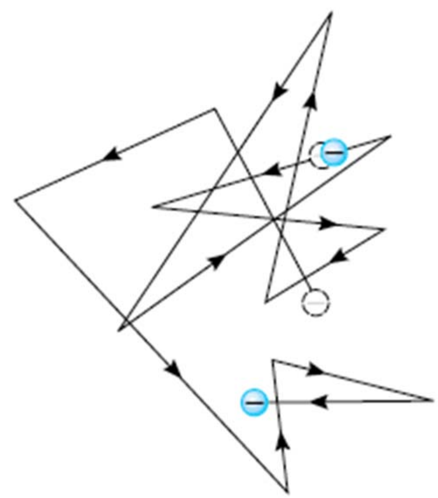
$$R = \frac{\rho}{2\pi L} \ln\left(\frac{b}{a}\right)$$

$$C = \frac{L}{2k_e \ln\left(\frac{b}{a}\right)}$$

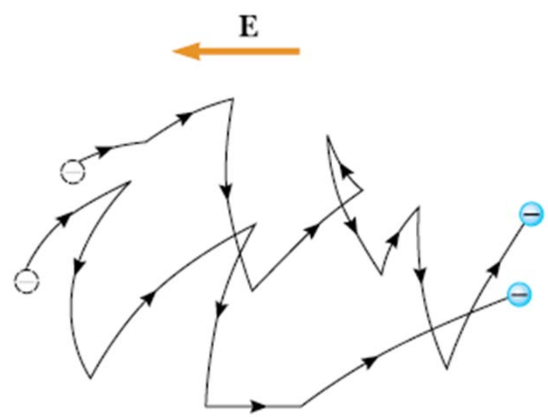
A Model for Electrical Conduction: The Drude Model



A regular array of atoms surrounded by a “cloud” of free electrons



Random movement under zero field



Random movement modified by a field

Drude Model

$$\mathbf{F} = q\mathbf{E} = m_e\mathbf{a}$$

$$\mathbf{a} = \frac{q\mathbf{E}}{m_e}$$

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = \mathbf{v}_i + \frac{q\mathbf{E}}{m_e}t$$

Now take the average over all times. Then $\mathbf{v}_i = 0$ (random movement),

$$\bar{\mathbf{v}}_f = \mathbf{v}_d = \frac{q\mathbf{E}}{m_e}\bar{t} = \frac{q\mathbf{E}}{m_e}\tau$$

, where τ is the mean time between collisions

$$J = nqv_d = \frac{nq^2E}{m_e}\tau$$

$$J = \sigma E$$

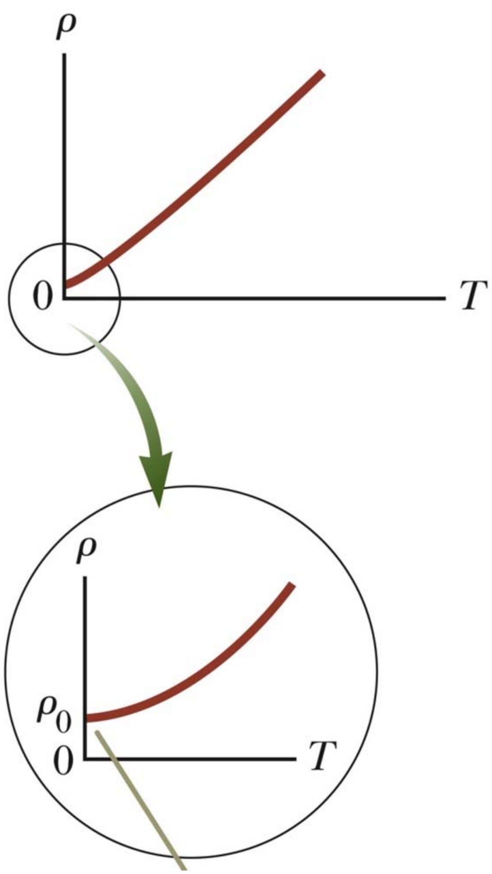
$$\boxed{\sigma = \frac{nq^2\tau}{m_e} \qquad \rho = \frac{m_e}{nq^2\tau}}$$

$$l = \tau \cdot v_d$$

is the mean free path

Resistivity and Temperature

Resistivity in metals is linear with temperature over a limited range



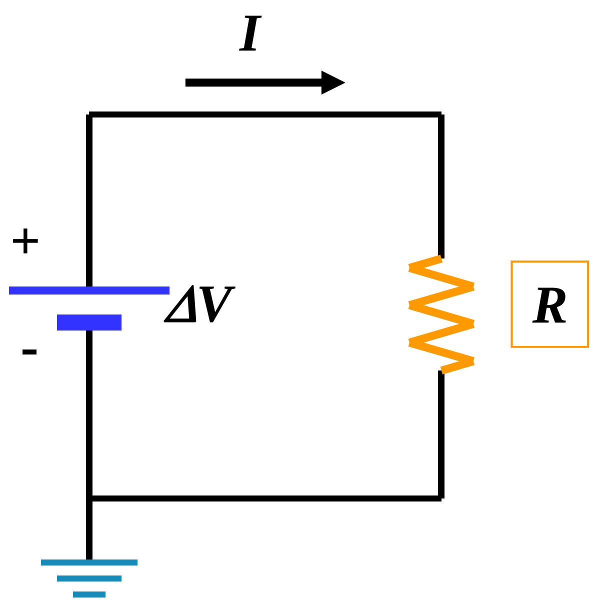
$$\rho = \rho_0\left[1 + \alpha\left(T - T_0\right)\right]$$

$$R = R_0\left[1 + \alpha\left(T - T_0\right)\right]$$

α : temperature coefficient of resistivity

$$\alpha = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T} \quad (C^{-1})$$

Electrical Power



$$\Delta U_{battery} = \Delta Q \Delta V$$

$$\Delta U_{resistor}(heat) = \Delta U_{battery}$$

$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q \Delta V}{\Delta t} = I\Delta V$$

$$\mathcal{P} = I\Delta V$$

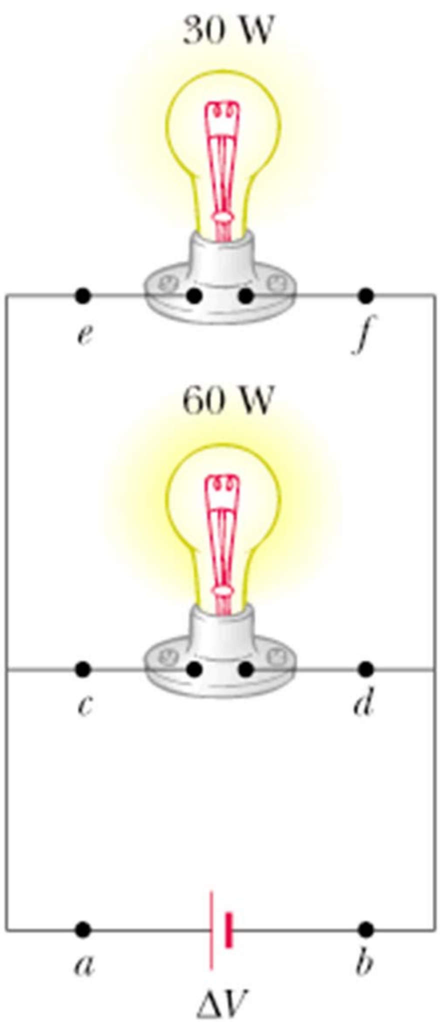
Electrical Power
V.A = Watts(W)


Symbol of a Resistor

$$\mathcal{P} = I^2 R = \frac{(\Delta V)^2}{R}$$

Power Dissipated
on a Resistor

Rank the Currents



$I_a = I_b$

$I_c = I_d$

$I_e = I_f$

$I_a = I_c + I_e$

$\mathcal{P}_{60W} > \mathcal{P}_{30W}$

$I_c > I_e$

$$I_a = I_b > I_c = I_d > I_e = I_f$$

Summary

- Current is the net rate of charge flow.
- Electrons move at the drift velocity.
- Resistance is the ratio of voltage applied to current. The ratio is linear for most metals.
- Resistivity is a material property.
- Electrical energy will be converted to thermal energy on a resistor. The rate of conversion is the power.

For Next Class

- Reading Assignment
  - Chapter 28 – Direct Current Circuits
- WebAssign: Assignment 5